A Systematic Approach to the Problem of Equivalent Circuit Model Generation

T. Mangold, P. Russer
Institut für Hochfrequenztechnik, Technische Universität München,
Arcisstraße 21, D-80333 München, Germany

Abstract

We present a general method for the generation of lumped element equivalent circuits for linear passive reciprocal distributed microwave circuits from time domain scattering parameters. The method was applied to model a Wilkinson power divider. The applicability of the generated circuit model in transient network simulation is demonstrated.

1 Introduction

While trends in the microwave and VLSI technology are to increase operating frequencies and digital clock rates, problems of signal integrity are becoming more and more the limiting factors in modern circuit design. To overcome restrictions caused by interconnect effects like crosstalk and signal distortion accurate full-wave analysis based modeling techniques have to be applied. Distributed interconnects are normally part of complex systems including lumped nonlinear subcircuits. Therefore, network based simulation techniques usually used for modeling lumped subcircuits have to be combined with full-wave analysis methods. One possible way is to directly integrate lumped elements into a full-wave analysis [2, 6]. However, to reduce the modeling effort and increase efficiency it is advantageous to separate EM and network simulation combining the single simulation results afterwards [7]. The most common method realizing this concept is the substitution of distributed circuit elements by lumped element equivalent circuit models. Though there are libraries of equivalent circuits models included in almost every microwave design CAD tool, only a few papers found in literature address the problem of a systematic method for the generation of lumped element equivalent circuits. In netlist extraction methods utilizing system identification techniques controlled stability and passivity of the generated circuit model is missing [3]. In [8] a straightforward parameter extraction technique for linear reciprocal lossless distributed circuits is given.

In this work we make use of this technique and its extension to lossy structures given in [5] presenting a method for an automated, computer aided generation of lumped element equivalent circuits for linear reciprocal distributed microwave circuits. Resulting circuit models conserve all basic circuit properties of a modeled distributed microwave circuit like passivity and reciprocity. The method is based on a field theoretical analysis of the distributed multiport circuit by the time domain Transmission Line Matrix (TLM) method. Topology as well as parameters of the lumped element equivalent circuit are generated after specifying an arbitrary but finite interval of frequencies.

2 Model Generation

Starting with a three-dimensional electromagnetic full-wave analysis of a distributed multiport we obtain the impulse response functions for reflection and transmission between the ports. For this we use the time domain TLM scheme with symmetrical condensed nodes [4]. An irregularly graded mesh reducing the computational effort on the one hand and providing an efficient way for exact boundary positioning on the other is applied [1]. Post-processing of simulation results and deembedding is done prior to the actual model generation process. Based on a canonical representation of the admittance matrix $\mathbf{Y}(p)$ of a linear reciprocal lossless two-port an equivalent circuit model may be specified [9]. We have extend this method to lossy linear reciprocal multiports [5]. The admittance matrix may be represented by

$$\mathbf{Y}(p) = \mathbf{A}^{(0)} + + \sum_{n=1}^{N} \left(\frac{A_0^{(n)}}{p-\alpha_n} + \frac{A_0^{(n)*}}{p-\alpha_n^*} \right) \cdot \mathbf{A}^{(n)} + \mathbf{A}^{(\infty)} p \quad (1)$$

where $\mathbf{A}^{(n)}$ are real, symmetric and positive semidefinite matrices. In order to establish lumped element equivalent circuits from time domain scattering signals in a systematic way, the process of model generation is splitted into subsequent steps:

- First, the locations and complex amplitudes of N admittance function poles α_n are extracted directly from the multiports admittance parameters $\mathbf{Y}(p)$ after specifying an arbitrary, but limited frequency range of validity. The Laplace transforms $\mathbf{Y}(p)$ are calculated numerically from the impulse response functions of the multiport. The complexity of the resulting lumped element equivalent circuit is determined by the number of poles included into the model.
- Pole amplitudes and phases are mapped into the valid parameter space for passive multiports, which is defined by

  $$\mathrm{Re}\left\{A_0^{(n)}\right\} > 0\,, \left|\arg\left\{A_0^{(n)}\right\}\right| \le \left|\arg\left\{-\jmath\alpha_n\right\}\right| \\ \mathrm{Im}\left\{A_0^{(n)}\right\} \cdot \mathrm{Im}\left\{\alpha_n\right\} \ge 0 \quad (2)$$

  This is achieved while clipping all negative eigenvalues of resulting matrices $\mathbf{A}^{(n)}$ to zero.
- The best valid approximation in the sense of a chosen error function is generated by fitting (1) to the corresponding frequency domain admittance parameters while maintaining positive semidefinite matrices $\mathbf{A}^{(n)}$.
- Afterwards all matrices $\mathbf{A}^{(n)}$ are decomposed into sums of symmetrical rank 1 matrices.
- Finally, the lumped element equivalent model of the analyzed multiport circuit may be constructed explicitly.

We are using a gradient based steepest-ascent search algorithm for the extraction of pole locations from the absolute value of admittance parameters $Y_{ij}(p)$. This requires a set of starting points converging towards the N most relevant poles. It can be taken from contour plots of the examined admittance parameter by inspection [5]. The two major drawbacks of this possible method are the facts, that it cannot be automated easily and that due to the limited resolution of contour plots poles may be omitted.

To circumvent these problems, a modified pole extraction algorithm was developed. In a first step, we use scalar model functions $M_{ij}^N(p)$ derived from (1).

$$M_{ij}^N(p) = A^{(0)} + + \sum_{n=1}^{N} \left(\frac{A^{(n)}}{p-\alpha_n} + \frac{A^{(n)*}}{p-\alpha_n^*} \right) + A^{(\infty)} p \quad (3)$$

Approximating admittance parameters $Y_{ij}(p)$ by $M_{ij}^N(p)$ is done without respect to the mentioned parameter half space of passive multiports. Instead of extracting poles from the goal functions $Y_{ij}(p)$, the steepest-ascent search algorithm is applied to a new function $D_{ij}(p)$, which is defined by the difference between $Y_{ij}(p)$ and the present model function $M_{ij}^N(p)$ containing all already extracted poles α_n. $D_{ij}(p)$ doesn't contain these poles anymore and therefore ensures the convergence to a different pole location $\alpha \neq \alpha_n|_{n\in[1,N]}$. After a new pole location α has been found, its corresponding complex amplitude $A^{(n)}$ has to be determined. This can be done analytically via the following approximation, which holds in the close neighborhood $|p-\alpha_i| < \epsilon$ of a function pole α_i.

$$Y(p) = A^{(0)} + \sum_{n=1}^{N} \left(\frac{A^{(n)}}{p-\alpha_n} + \frac{A^{(n)*}}{p-\alpha_n^*} \right) + \mathbf{A}^{(\infty)} p \\ \approx \frac{A^{(i)}}{p-\alpha_i} + \frac{A^{(i)*}}{p-\alpha_i^*} \quad (4)$$

Constructing an l_2-error minimization problem for a test set $T = \{p_t : |p_t - \alpha_i| < \epsilon\}$ makes the solution robust against numerical errors. It leads to

$$\mathrm{Min} \left[\sum_{t=1}^{N_t} \left| \frac{A^{(i)}}{p_t - \alpha_i} + \frac{A^{(i)*}}{p_t - {\alpha_i}^*} - Y(p_t) \right|^2 \right] \quad (5)$$

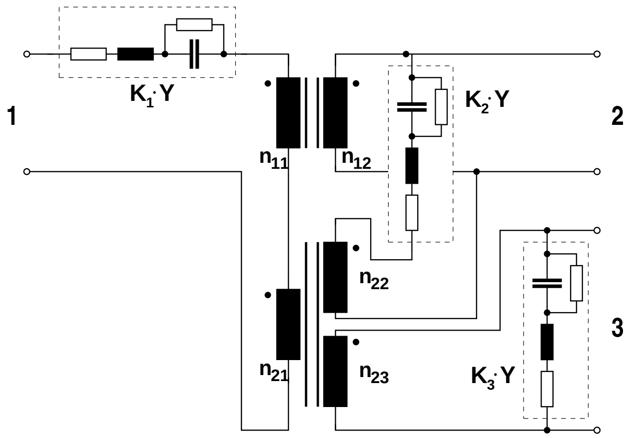

Figure 1: A compact three-port element contributing a full rank admittance matrix

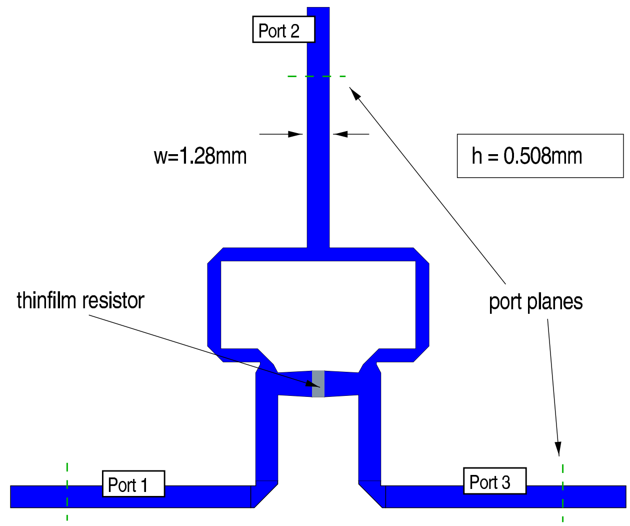

Figure 2: Modeled Wilkinson power divider ; design frequency = 4 GHz

which can be solved in closed form. While choosing the global maximum of $D_{ij}(p)$ within the frequency range of validity as starting values, the N most relevant poles α_n are automatically found by an iterative procedure specified in such a manner.

Obtained results for complex pole amplitudes $A^{(n)}$ might not be located within the valid parameter space of (1). This is mainly caused by simulation errors contained in the time domain scattering signals used for the calculation of multiport admittance parameters $Y_{ij}(p)$. However, generation of an absolute stable lumped element equivalent circuit model requires a valid parameter set of (1). Therefore, all pole locations α_n are fixed and pole amplitudes are mapped to the closest point of the valid parameter space. Using this parameter set as a starting value for fitting (1) to the actual multiport admittance matrix $\mathbf{Y}(p)$ leads to the final parameter set used for circuit model generation. A lumped element equivalent circuit model for the distributed multiport can then be specified directly according to the method given in [5]. It is built of N shunt connected compact three-port elements as depicted in Fig1.

3 Example

To demonstrate the versatility of the proposed method a distributed three-port circuit has been modeled. The geometry of the Wilkinson power divider is depicted in Fig.2. It is built in microstrip technique on a $0.508\,$mm thick polyimid substrate with $\epsilon_r = 3.0$. In the TLM simulation of the structure metalization and thin-film resistor were treated as infinitely thin. The design frequency of the power divider is 4 GHz and a conductor width of $1.28\,$mm gives port impedances of $50\,\Omega$. Comparing the simulation results in Fig.3 with measurement data shows good agreement. For example at design frequency: simulated transmission $S_{21} = -3.19$[dB] compared to -3.07[dB] measured; $S_{31} = -23.2$[dB] compared to -21.4[dB] measured. We have chosen a broad frequency range of validity for demonstration purpose. It was specified to range from DC up to 18 GHz. The most simple circuit model representing the frequency characteristic over the whole band needs at least 12 poles to be included. Making use of the automated pole extraction algorithm, we also generated a more accurate lumped element equivalent circuit model including 47 poles. Caused by the symmetry of the structure most poles are only associated with port 1 and 3 reducing the complexity of the model. A comparison between an extra TLM simulation limited to the bandwidth of the circuit model and a SPICE simulation of the generated equivalent circuit model are shown in Fig.4. Remaining deviations are caused by an optimization of admittance and not scattering parameters.

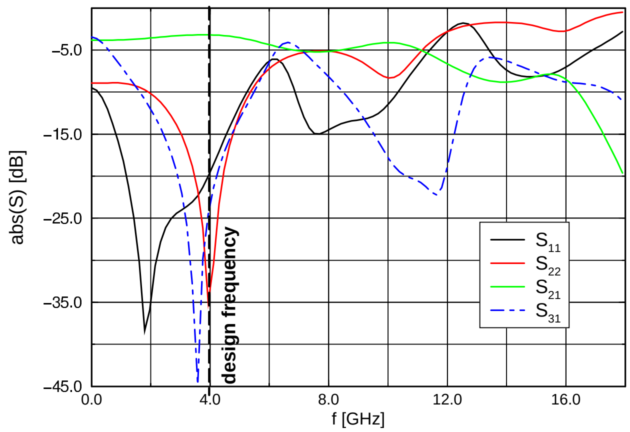

Figure 3: Scattering parameters of the power divider obtained from TLM simulation

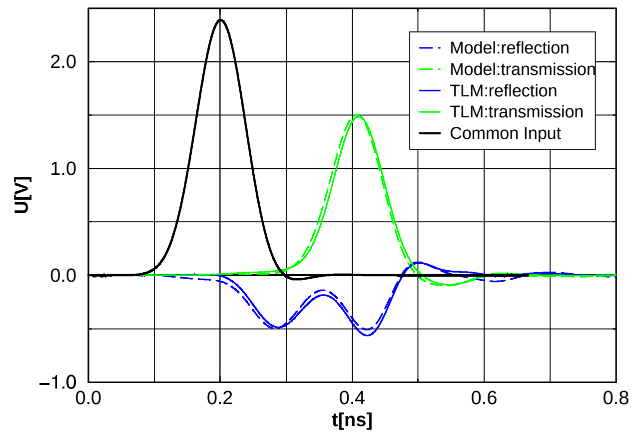

Figure 4: Full-wave analysis (TLM simulation) and an equivalent SPICE simulation of the generated equivalent circuit model; pulse bandwidth = 18 GHz; excitation at port 2

4 Conclusion

We have presented a computer aided method for the generation of lumped element equivalent circuit models for distributed microwave components based on time domain scattering signals. It produces topology as well as parameters of a model conserving basic properties like reciprocity and passivity. The new pole extraction algorithm allows an almost complete automation the method. A Wilkinson power divider has been modeled and the applicability of the generated lumped element equivalent circuit in transient spice simulation has been demonstrated.

Acknowledgment

This work has been financially supported by the German Federal Ministry of Education, Science, Research and Technology (BMBF).

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